# Counterparty Credit Exposure in the Presence of Dynamic Initial Margin

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Download source code from http://modval.org/models/mpr/aps/

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## Overview

- Even under full collateralization, considerable exposure remains due to the Margin Period of Risk (MPoR).
- BCBS-IOSCO dynamic bilateral initial margin (IM) was introduced to eliminate this residual exposure and CVA.
- We show that due to the way ISDA/CSA operate, dynamic IM is not nearly as effective in doing so as it was previously thought to be.
- In a highly stylized classical model for MPoR, residual CVA under IM is around 1% of CVA without IM.
- Once the precise legal terms of ISDA/CSA are incorporated into the model, depending on the portfolio this number could be as high as 20% to 50%

# 1. Exposure under Variation Margin

## Delays in normal exchange of collateral

- The amount of collateral held by the parties is adjusted via the mechanism of a margin call.
- Margin call is a chain of events which takes several days to complete.
- With daily re-margining, several such chains are running concurrently
- This means that the changes in variation margin are always running behind the changes in portfolio value.
- This lag arises due to inevitable operational and legal delays that are “baked into” the workings of the ISDA/CSA contracts that govern OTC trading.

## Settlement risk during the period in advance of default

- The first payment type (*trade flows*) covers the contractual cash and asset flows defined in the trade's term sheet
  - A missed trade flow is a serious event, and a "failure to pay" can rapidly lead to default and trade termination unless cured promptly.
- The second payment type (*margin flows*) is the exchange of collateral between the parties
  - ISDA/CSA affords relatively mild treatment to a party who misses a margin flow.
  - Indeed, partially missing a margin payment is a common occurrence, as disputes about margin amounts happen regularly (and can sometimes persist for years).

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## Collateralized exposure

- Consider trade portfolio at time $t$ from the viewpoint of the non-defaulting party $B$ ("bank", or "us")
- Let $K(t)$ denote the collateral that $B$ can *actually* rely on for portfolio termination; this amount very likely will differ from the CSA-stipulated amount due to margin transfer time lags and some degree of non-performance by the defaulting party $C$.
- In addition, it is possible that some trade flows were missed; let us denote their value at time $t$, including accrued interest, as $\mathrm{UTF}(t)$.
- Then we may redefine exposure as

$$E(t) = \left(V(t) + \mathrm{UTF}(t) - K(t)\right)^{+}. \quad (1)$$

## Collateralized exposure

- We also define time-0 expectation of future time-$t$ exposure as
$$\mathrm{EE}(t) = \mathbb{E}_0(E(t)),$$
where $\mathbb{E}$ is the expectations operator in a relevant probability measure.

## Margin period of risk (MPoR)

- Because of delays in normal exchange of collateral as well as settlement risk of margin and trade flows, it is well recognized that credit default cannot be treated as a one-time event for the purposes of modeling exposure.
- Rather, the entire sequence of events leading up to and following the default must be considered, from the market observation date for the last successful margin call to the time when the amount of loss becomes known (in industry parlance, “crystallized”).
- These events unfold the time period called the *margin period of risk* (MPoR).

# 1. Exposure under Variation Margin

## The Classical model for MPoR is highly stylized

- In the Classical model for exposure, the counterparties are assumed to have oddly synchronized behavior inside the MPoR.
  - For instance, one common model variant (which we denote Classical−) assumes that margin and trade flows by both counterparties terminate simultaneously, at the beginning of MPoR.
  - Another approach (Classical+) assumes that margin flows terminate at the beginning of the MPoR, but trade flows terminate (simultaneously) at the end of the MPoR.
- Surprisingly, the Classical+ and Classical− approaches continue to co-exist in the market, and neither have become the sole market practice.

## Shortcomings of the Classical model

- In practice, trade flows are paid at least by $B$ in the beginning of the MPoR, and are not paid by at least $C$ at its end.
- For instance, there is typically at least a 2-3 business day lag between the start of the MPoR (the market observation date for the last full margin payment) and the date when $B$ definitively observes that $C$ has missed paying a margin flow; during this period $B$ would make payments
- During the last 2-3 days of the MPoR just prior to termination, $C$ has already defaulted and neither party is likely making trade payments.

# 1. Exposure under Variation Margin

# Taking into account the legal and practical aspects of ISDA/CSA

- The Advanced model represents MPoR using four model parameters, instead of one in the Classical model:
  1. Time $\delta_C$ and $\delta_B$ between the last market data observation date for which the margin flow is paid as prescribed by $C$ or $B$, and the end of MPoR
  2. Time $\delta'_C$ and $\delta'_B$ between the last settlement date on which trade flow is paid as prescribed by $C$ or $B$, and the end of MPoR
- Note that the proposed model parametrization includes Classical$+$ and Classical$-$ models as limit cases.

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# Model parametrization

- In our model's timeline we recognize that $B$ may take the action of stopping margin and trade flows at a different time than $C$ does.
- The end of the MPoR is defined the same way as the classical model, to coincide with ETD.

| Event | Notation |
|---|---|
| Observation date for the last margin flow by $C$ | $t_C = t - \delta_C$ |
| Observation date for the last margin flow by $B$ | $t_B = t - \delta_B$ |
| Date of last trade flow payment by $C$ | $t'_C = t - \delta'_C$ |
| Date of last trade flow payment by $B$ | $t'_B = t - \delta'_B$ |
| ETD | $t$ |

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## Aggressive and Conservative calibration options

- Aggressive calibration assumes maximally stringent enforcement of legal rights
- Conservative calibration may be used when a degree of leniency in enforcement is expected (e.g. for clients), or the bank is not able to enforce its rights to the full extent for operational reasons.
- Aggressive and Conservative parameter choices may be used as as two limit scenarios for materiality and model risk analysis.

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## Calibration parameters

| Parameter | Conservative | Aggressive | Classical$+$ | Classical$-$ |
|---|---|---|---|---|
| $\delta_C$ | 15bd | 7bd | 10bd | 10bd |
| $\delta_B$ | 9bd | 6bd | 10bd | 10bd |
| $\delta'_C$ | 8bd | 4bd | 0bd | 10bd |
| $\delta'_B$ | 3bd | 4bd | 0bd | 10bd |

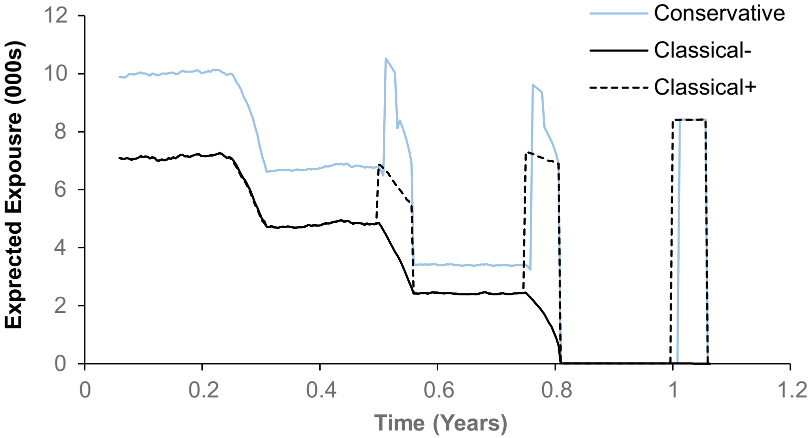
Notes:

- Conservative calibration includes trade flow gap: time period where only $B$ pays trade flows (Herstatt risk).
- Both Aggressive and Conservative calibrations include margin flow gap: time period where only $B$ pays margin flows.

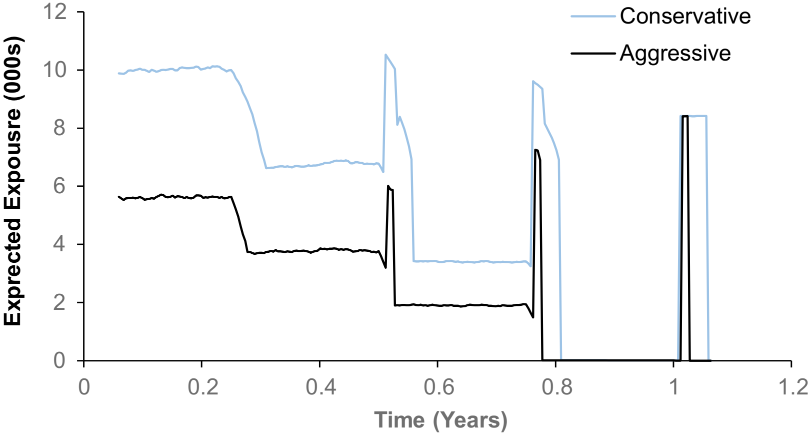
## Examples setup

- To gain intuition for our model, we present exposure profiles and CVA metrics for several trade and portfolio examples, using Classical$+$, Classical$-$, Aggressive and Conservative calibrations.
- Model parameters and source code are available at http://modval.org/models/mpr/ and http://modval.org/papers/aps2016/

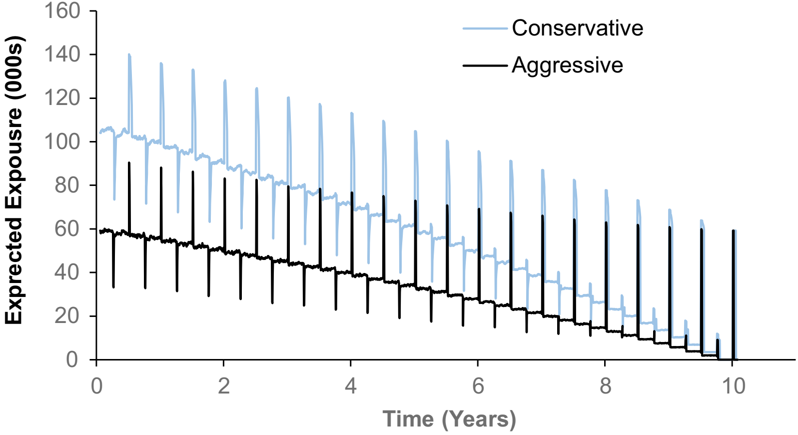
# 1y single currency swap, Classical vs Advanced Model



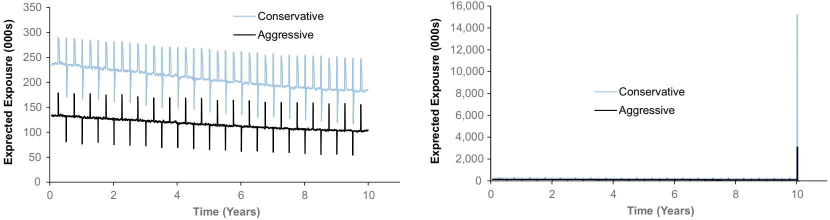
# 1y single currency swap, Advanced Model



# 10y single currency swap, Advanced Model

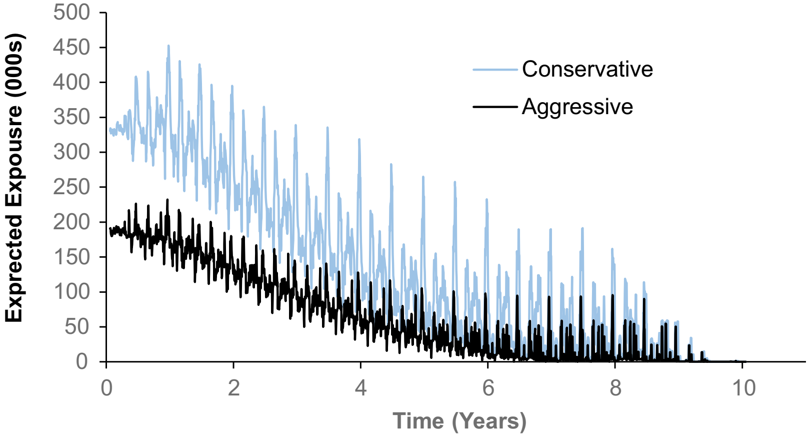


# 10y XCCY swap, Advanced Model

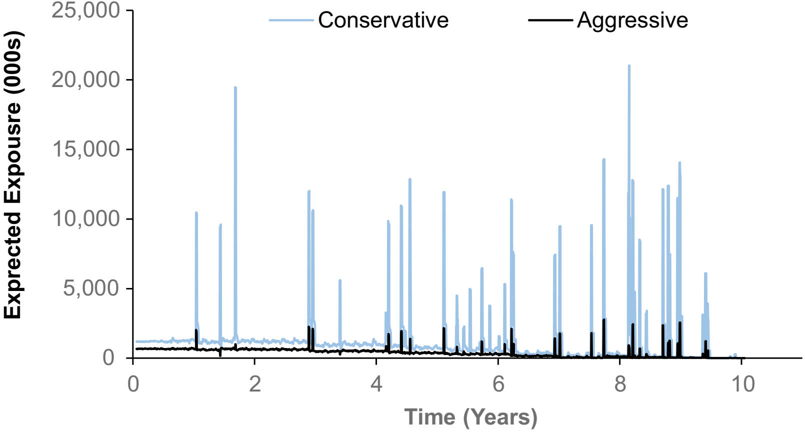



**(a)** x50 reduced size **(b)** Full scale

## Single currency swap portfolio, Advanced Model



# XCCY swap portfolio, Advanced Model



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## Impact on CVA

| Securities | Conservative | Classical15– | Aggressive | Classical7– |
|---|---|---|---|---|
| 10Y Swap | 7,785 | 5,242 | 4,187 | 3,561 |
| Swap Portfolio | 29,399 | 17,961 | 14,459 | 12,246 |
| 10Y XCCY Swap | 18,131 | 10,122 | 8,524 | 6,889 |
| XCCY Swap Portfolio | 167,066 | 56,807 | 49,825 | 38,785 |

Notes:

- Classical15– has 15bd MPoR and Classical7– has 7bd MPoR to match the respective Advanced model.
- Settlement risk contribution to CVA is comparable or greater than the contribution of market factor evolution.
- CVA for the XCCY portfolio is particularly high for the Conservative calibration due to Herstatt risk.

# 2. Exposure under Variation and Initial Margin

## BCBS-IOSCO dynamic initial margin

- In 2013, the BCBS and IOSCO issued BCBS-261 document, under which any two covered entities that are counterparties in non-centrally cleared derivatives are required to both exchange variation margin (VM) under a zero threshold margin agreement; and post initial margin (IM) to each other without netting the amounts.
- Initial Margin is applied on top of Variation Margin in order to (nearly) eliminate the residual exposure due to MPoR
- We will soon see that IM fails to do so if settlement risk of margin and trade flows are taken into account.

# Exposure under both VM and IM

- For a single netting set covered by VM and IM, B's exposure to C can generally be represented as:

$$E(t) = [V(t) + \mathrm{UTF}(t) - \mathrm{VM}(t) - \mathrm{IM}(t)]^+ \tag{2}$$

where

- $V(t)$ is the netting set value at time $t$
- $\mathrm{UTF}(t)$ is the net value of all trade flows (TF) scheduled to be paid prior to the closeout time $t$, but actually unpaid by either C (positive) or B (negative), accrued to $t$
- $\mathrm{VM}(t)$ is the variation margin actually available to B at the closeout time $t$
- $\mathrm{IM}(t)$ is the initial margin actually available to B at the closeout time $t$

# BCBS-IOSCO dynamic initial margin rules

- Under the BCBS and IOSCO rules, regulatory IM can be calculated either by an internal model or by look-up in a standardized schedule.
- If an internal model is used, the IM at the netting set level is 99% VaR for 10 day horizon in case of daily remargining
- Diversification across distinct asset classes is not recognized, and the IM internal model must be calibrated to a period of stress for each of the asset class.
- The required levels of the IM will change as cash flows are paid, new trades are booked or markets move.

# Estimation of future initial margin levels

- It is difficult to calculate future IM levels if one wants to incorporate all the restrictions and "twists" of the IM rules: stress calibration, limited diversification allowance, and, for CCPs, add-ons for credit downgrades and concentration risk.
- For the purposes of estimating the effect of IM on CVA, we will substitute model VaR for the BCBS-IOSCO prescribed VaR.

## Calculation method

- Assuming that under our valuation model portfolio values are locally Gaussian, IM levels depend on future local volatility of the portfolio value for the period $[t-\delta_C, t]$ estimated at $t-\delta_C$ along each path.
- Denoting the IM horizon by $\delta_{\mathrm{IM}}$ and the local volatility of portfolio value at time $u$ on path $m$ via $\sigma_m(u)$, the IM available to $B$ at the ETD date $t$ on path $m$ is given by

$$\mathrm{IM}_m(t-\delta) = \sigma_m(t-\delta)\sqrt{\delta_{\mathrm{IM}}}\Phi^{-1}(q) \tag{3}$$

where $q$ is a confidence level (often 99%) and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution function.

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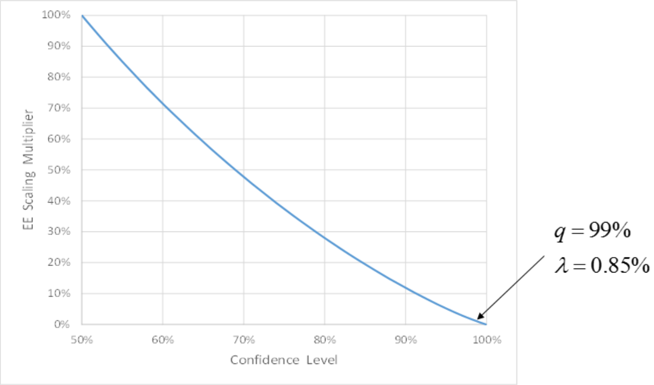
## Scaling approach to computing exposure under both VM and IM away from trade flows

- The most straightforward approach is to use regression techniques to estimate IM on each simulation path and then subtract it from the exposure realized on that path
- Because the IM confidence level is so high ($q = 99\%$), only very few paths (1% on average) would result in non-zero exposure.
- Instead, we will compute EE without IM, and then apply a pre-computed scaling multiplier $\lambda$ to get EE with IM
- If portfolio process is locally Gaussian, the scaling multiplier depends only on the IM confidence level $q$ and the ratio of the IM horizon $\delta_{\mathrm{IM}}$ to the MPoR length $\delta_C$

# Scaling multiplier

- When $\delta_{IM} = \delta_C$, multiplier $\lambda$ is given by

$$\lambda = \left[\phi\left(\Phi^{-1}(q)\right) - (1-q)\,\Phi^{-1}(q)\right] / \phi(0)$$

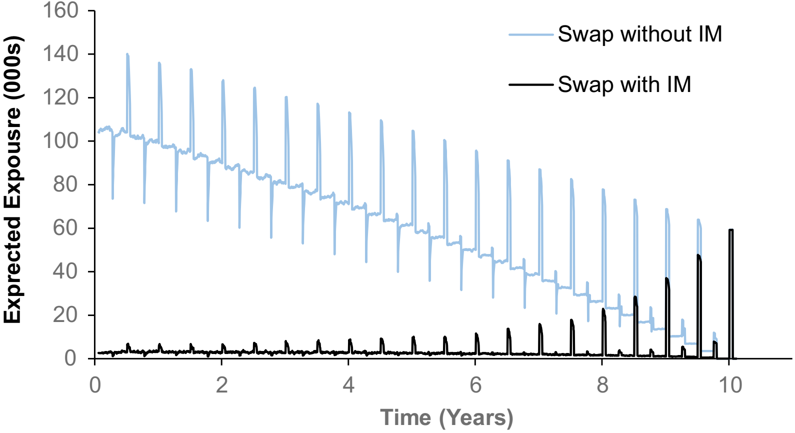


## Does IM (nearly) eliminate the residual exposure?

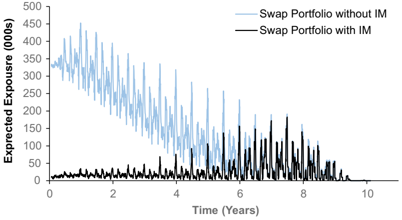
- In the absence of trade flows within MPoR, the reduction in expected exposure (EE) is indeed dramatic: it is reduced by a factor of over x100 when IM is defined as 99% VaR.
- This is why it is widely believed that IM will practically eliminate the residual exposure
- However, the reduction of the EE is much less dramatic when there are trade flows within MPoR
- Scenarios where e.g. B makes a trade payment to C within the MPoR and does not get the variation margin back may result exposure peaks in excess of IM, resulting in a counterparty credit loss

# 2. Exposure under Variation and Initial Margin

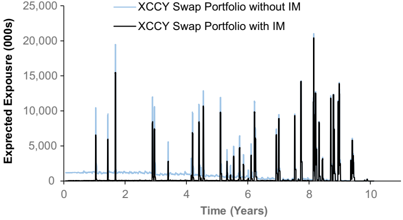
# 10y Vanilla swap IM



# Portfolio IM




**(a)** Single currency swap portfolio, 16% residual CVA




**(b)** Cross currency swap portfolio, 34% residual CVA

## Summary

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## Performance bottleneck with brute force method on a daily grid

- In exposure calculations for realistic portfolios, horizons can be very long, often exceeding 30 years.
- For such lengthy horizons, brute-force Monte Carlo simulation of exposures on a daily, or even weekly, time grid will often be prohibitively slow.
- Coarsening of the time grid will inevitably fail to capture both the “worst case” margin effect and the trade spikes that are key to accurately exposure.
- We will now discuss ways to capture exposure without having to resolve to brute-force daily simulation.

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# The coarse grid lookback method

- Assume that portfolio simulation is not done daily, but instead only on a coarse grid $\{s_j\}$
- To achieve acceptable computational performance, the time step of the coarse model grid, $s_j - s_{j-1}$, must be significantly greater than the length of the MPoR.
- This, however, would preclude one from establishing the portfolio value at $s_j - \delta$.
- The Coarse Grid Lookback method deals with this issue by simply adding a second ”lookback” time point $s_j - \delta$ to all “primary” measurement times $s_j$, in effect replacing each node of the coarse model grid by a pair of closely spaced nodes.

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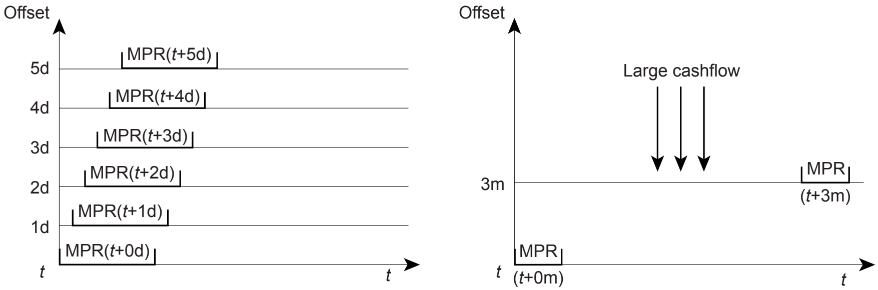
# Coarse grid lookback method diagram



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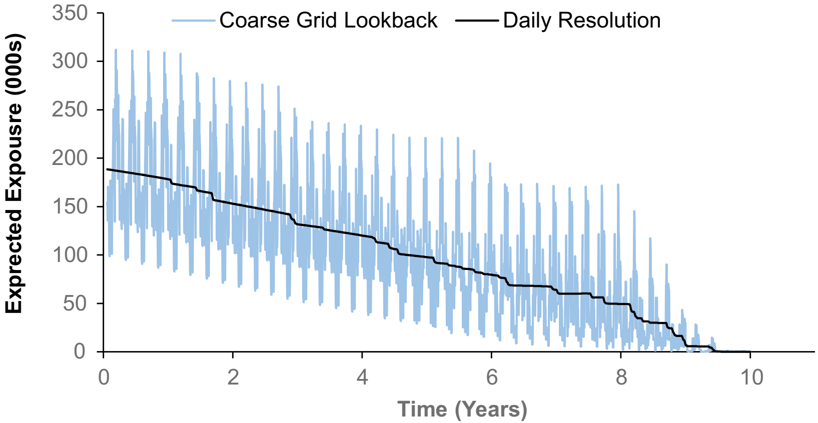
## CVA P&L using coarse grid lookback method

- To illustrate the problem with the coarse grid lookback method, let us define the concept of time $t$ “forward” CVA, denoted $\mathrm{CVA}^t$, which is the risk neutral projection of CVA at a future time $t$.
- As CVA is an integral of exposure, spikes in exposure profile should result in jumps rather than oscillations in $\mathrm{CVA}^t$.
- When one of the “MPoR windows” moves over a large trade flow, the contribution to CVA temporarily increases
- This results in large oscillations of $\mathrm{CVA}^t$ whose presence is highly unattractive when CVA is computed and reported as part of daily P&L.

# Forward CVA for a Portfolio of 50 Cross-Currency Swaps, Classical+ model



## Brownian Bridge acceleration method

- Re-pricing the entire portfolio daily at each simulation path is prohibitively expensive for large portfolios.
- On the other hand, merely simulating risk factors at a daily resolution is generally feasible, as the number of the simulated risk factors is typically relatively small (e.g., several hundred)
- Having produced risk factor paths on a daily grid, one can normally also produce realized trade flows, as they are simple functions of the realized risk factors.
- The Brownian bridge algorithm overlays ”diffusion” approximated by the Brownian Bridge between coarse model grid nodes with the realized trade flows computed from the first principles on their precise dates.

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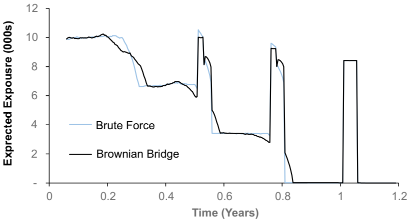
# Numerical example for the accelerated method




**(a)** Coarse grid nodes far from cashflow dates

**(b)** Coarse grid nodes near cashflow dates

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